

The Non-Minimal Supersymmetric Standard Model at Large $\tan \beta$

B. Ananthanarayan

Institut für Theoretische Physik, Universität Bern,
CH 3012, Bern, Switzerland

and

P. N. Pandita

Universität Kaiserslautern, Fachbereich Physik,
Erwin-Schrödinger-Strasse, D-67663 Kaiserslautern, Germany

and

Department of Physics, North Eastern Hill University,
Shillong 793 003, India*

Abstract

We present a comprehensive analysis of the Non-Minimal Supersymmetric Standard Model (NMSSM) for large values of $\tan \beta$, the ratio of the vacuum expectation values of the two Higgs doublets, which arise when we impose the constraint of the unification of Yukawa couplings in the model. In this limit we show that the vacuum expectation value of the singlet is forced to be large, of the order of 10 TeV. The singlet decouples from the lightest CP-even neutral Higgs boson and the neutralinos. We compare our results with the corresponding particle spectrum of the Minimal Supersymmetric Standard Model in the same limit. With the exception of the lightest Higgs boson, the particle spectrum in the model turns out to be heavy. The Higgs boson mass, after the inclusion of radiative corrections, is found to be in the neighbourhood of ~ 130 GeV.

* Permanent address

1 Introduction.

The recent determination of the coupling constants of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ model of strong and electroweak interactions is compatible with the supersymmetric unification [1] of these couplings with a supersymmetry [2] breaking scale of the order of 1 TeV. In supersymmetric (SUSY) models, at least two higgs doublets (H_1 and H_2) with opposite hypercharge are required to give masses to the up- and down- type quarks (and charged leptons), and to cancel gauge anomalies. The ratio of the vacuum expectation values of the neutral components ($\langle H_1^0 \rangle = v_1$ and $\langle H_2^0 \rangle = v_2$) of these two Higgs doublets, $\tan \beta \equiv \frac{v_2}{v_1}$, is a crucial parameter for the predictions from supersymmetric grand unified theories. One particularly predictive framework is based on the assumption that the heaviest generation fermions lie in a unique **16**-dimensional representation of the unifying gauge group $SO(10)$, with the Higgs doublets in a **10**-dimensional representation of the group [3]. Furthermore, if one makes the additional assumption that the fermion masses are generated by a single complex **10**-plet of $SO(10)$ through a $h \cdot \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10}$ term in the superpotential at the unification scale M_X , determined from gauge coupling unification, one would have the prediction [3]:

$$h_t = h_b = h_\tau = h \tag{1.1}$$

where $h_{t,b,\tau}$ are the Yukawa couplings of t , b and τ . The coupled system of gauge and Yukawa couplings are then evolved from M_X down to the weak scale, and $\tan \beta$ is determined from the accurately measured value of $m_\tau = 1.78$ GeV. When h is chosen in such a manner as to yield a value for $m_b(m_b)$ in its observed range of 4.25 ± 0.10 GeV [4], a rather good prediction for the top-quark mass is obtained, which, with the present central value of $\alpha_S(M_Z) = 0.12$, lies in the range favored by the experimental data [5]. In such a situation, $\tan \beta$ is found to saturate what is considered to be a theoretical upper bound

on its values of m_t/m_b , and the Yukawa coupling h is found to come out to be rather large $O(1-3)$, with a certain insensitivity to the exact value, since it is near a fixed point of its evolution.

At the weak scale, the minimal particle content of such a grand unified theory is that of the minimal supersymmetric standard model [2], with each bosonic (fermionic) degree of freedom of the standard model complemented by fermionic (bosonic) one, apart from having two Higgs doublets as mentioned earlier. However, it does not involve in any great detail the remaining aspects of the embedding of the standard model into a supersymmetric grand unified framework. With the particle content of the MSSM and an additional discrete symmetry, called R-parity [2], which forbids couplings that can lead to rapid nucleon decay, it is possible to construct a self-consistent and successful framework. In particular, the Higgs sector of the MSSM contains five physical degrees of freedom: two CP even (h^0 and H^0) and one CP-odd (A^0) neutral and complex charged Higgs bosons (H^\pm). The spectrum of the Higgs bosons is strongly affected by the grand unified theory assumptions such as the ones described above. For instance, the mass of the lightest higgs boson m_{h^0} , which is related through the D-term in the potential to the mass of the Z-boson (m_Z), after inclusion of radiative corrections is found to be $\lesssim 140$ GeV [6]. Furthermore, supersymmetry breaking is understood to arise from embedding the MSSM into a supergravity framework and writing down all the possible soft supersymmetry breaking terms consistent with gauge and discrete symmetries that define the model. It is usually assumed that many of the parameters describing these terms are in fact equal at the unification scale in order to have a predictive framework, which is motivated by such arguments as the weak principle of equivalence when applied to coupling of the SUSY breaking hidden sector to the known sector via gravitation. Such universal boundary conditions have had to be modified somewhat in order to avoid

fine tuning and to minimize the effects of possible radiative corrections to the b-quark mass induced through the supersymmetry breaking sector of the theory at the one-loop level [6, 7, 8, 9].

In $SU(5)$ type unification where $\tan\beta$ is free, the region $\tan\beta \simeq 1$ is also a region which is favoured for the unification of the b-quark and τ -lepton Yukawa couplings from the observed data [10]. One crucial difference between the two extremes is that for large values of $\tan\beta$, the Yukawa couplings of the b-quark (and that of the τ -lepton) always remain comparable to that of the top-quark, with the observed hierarchy in their masses arising from the large value of $\tan\beta$; when $\tan\beta \simeq 1$, the Yukawa couplings of the b-quark and that of the τ -lepton are much smaller than that of the top-quark.

Despite its many successes, it may be premature to confine our attention only to the MSSM, especially because of the presence of dimensionful Higgs bilinear parameter μ in the superpotential. An alternative to the MSSM that has been widely considered is the Non-Minimal Supersymmetric Standard Model (NMSSM), where the particle content of the Minimal Supersymmetric Standard Model (MSSM) is extended [11] by the addition of a gauge singlet chiral superfield S , and one in which dimensionful couplings are eliminated through the introduction of a discrete Z_3 symmetry. While the explicit coupling $\mu H_1 H_2$ is forbidden, an effective μ term is generated by the vacuum expectation value $\langle S \rangle (\equiv s)$ of the singlet Higgs field.

The NMSSM is characterised by the superpotential [12]

$$W = h_t Q \cdot H_2 t_R^c + h_b Q \cdot H_1 b_R^c + h_\tau L \cdot H_1 \tau_R^c + \lambda S H_1 \cdot H_2 + \frac{1}{3} k S^3, \quad (1.2)$$

where we have written the interactions of only the heaviest generation and the Higgs sector of the theory. In addition, one must add to the potential obtained from eq.(1.2), the most general soft supersymmetry breaking terms. These are:

$$(h_t A_t \tilde{Q} \cdot H_2 \tilde{t}_R^c + h_b \tilde{Q} \cdot H_1 \tilde{b}_R^c + h_\tau A_\tau \tilde{L} \cdot H_1 \tilde{\tau}_R^c + \lambda A_\lambda H_1 \cdot H_2 S + \frac{1}{3} k A_k S^3) + \text{h.c.}$$

$$+m_{H_1}^2|H_1|^2 + m_{H_2}^2|H_2|^2 + m_S^2|S|^2 + m_{\tilde{Q}}^2|\tilde{Q}|^2 + m_{\tilde{t}}^2|\tilde{t}_R^c|^2 + m_{\tilde{b}}^2|\tilde{b}_R^c|^2 + m_{\tilde{\tau}}^2|\tilde{\tau}_R^c|^2 \quad (1.3)$$

in the standard notation. In addition, there are soft SUSY breaking mass terms for $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ gauginos (λ' ; λ^a , $a = 1, 2, 3$; λ^i ; $i = 1, \dots, 8$), which we shall denote by M_1 , M_2 and M_3 , respectively. We note that if $k = 0$, the Lagrangian obtained from (1.2) has a global $U(1)$ symmetry corresponding to $N \rightarrow Ne^{i\theta}$, $H_1 H_2 \rightarrow H_1 H_2 e^{-2i\theta}$, which is broken by the vacuum expectation values (VEVs) of the Higgs fields. In order to avoid axion associated with this symmetry we require $k \neq 0$.

In this paper we present a study of the particle spectrum of the NMSSM, characterized by eq.(1.2) and eq.(1.3), for large values of $\tan\beta$ [13] in detail, and compare it with the corresponding spectrum in the MSSM. We carry out a renormalization group analysis of this model with universal boundary conditions and analyze the RG improved tree-level potential at the scale Q_0 . The cut-off scale for the renormalization group evolution is chosen to be the geometric mean of the scalar top quark masses which is roughly equal to the corresponding mean of the scalar b-quark masses as well, since the evolution of the Yukawa couplings of the t- and b-quarks are equal upto their hypercharges and the relatively minor contribution of the τ -lepton Yukawa coupling [we note here that this feature is crucial in justifying our use of the tree-level potential since with large $\tan\beta$, the Yukawa coupling of the b-quark is large, influencing the contributions of the large logarithms in the one-loop potential]. Whereas in the MSSM the parameters μ and B (the soft SUSY breaking parameter corresponding to the bilinear term in the superpotential of the MSSM) do not enter into the evolution of the other parameters of the model at the one-loop level, the situation encountered here is drastically different with a systematic search in the parameter space having to be performed with all parameters coupled from the outset. Our analysis of the minimization conditions that ensure a vacuum give rise to

severe fine tuning problems that are worse in the NMSSM in comparison to those in the MSSM. The problem is further compounded by our having to satisfy three minimization conditions in contrast to the two that occur in the MSSM. In studies of the model where $\tan\beta$ is a free and adjustable parameter, the tuning of parameters is possible in order to meet all the requisite criteria, viz., minimization conditions, requirement that the vacuum preserve electric charge and color, etc. However in the present case where $\tan\beta$ is fixed and large, what we find is a highly correlated system.

The plan of the paper is as follows: In Section 2, we review the basic framework of the MSSM at large $\tan\beta$ so as to set up the stage for our analysis of the NMSSM in the same limit, and thereby enable us to compare and contrast the two models. In Section 3 a detailed analysis of the NMSSM in this limit is presented. An important conclusion of our analysis is that the vacuum expectation value of the singlet is forced to be large in this limit. In Section 4 we present a numerical study of the model and the conclusions.

2 MSSM at Large $\tan\beta$

As mentioned in the introduction, in the minimal supersymmetric extension of the standard model, the ratio of the vacuum expectation values of the two Higgs doublets, $\tan\beta$, is an important parameter that determines the spectrum in an essential way. The unification of the third family Yukawa couplings, eq.(1.1), leads to $\tan\beta$ being determined in the MSSM, thus, reducing the parameter space significantly. The analysis of the MSSM starts with the scalar potential, where the parameters are evolved according to the one-loop renormalization group equations from M_X to the low energy scale Q_0 [6]. At tree-level it is possible to analytically find the minimum of the potential, and we simply quote the results here [14]. We start with the definitions:

$$\mu_1^2 = m_{H_1}^2 + \mu^2, \quad \mu_2^2 = m_{H_2}^2 + \mu^2, \quad \mu_3^2 = \mu B$$

where $m_{H_i}^2$, $i = 1, 2$ are the soft-supersymmetry breaking mass parameters for the Higgs fields, and B is the soft-supersymmetry breaking bilinear parameter corresponding to the $\mu H_1 H_2$ term in the superpotential of MSSM. In order that the minimum of the potential break $SU(2) \times U(1)$ gauge symmetry to $U(1)_{em}$, we must have $\mu_1^2 \mu_2^2 < \mu_3^4$, whereas to prevent the potential from being unbounded from below we require $\mu_1^2 + \mu_2^2 \geq 2|\mu_3|^2$ (note that the left-hand side of this is nothing but the square of the mass of the CP-odd Higgs boson, m_A^2). These requirements tend to favor a situation when only one of μ_1^2 or μ_2^2 is negative, or where both are positive and one is somewhat smaller than the other. From here we see that radiative electroweak symmetry breaking via the effects of h_t may be implemented in the MSSM in general [15], as it drives μ_2^2 to smaller (possibly negative) values in relation to μ_1^2 , while μ_3^2 can be easily set negative.

In the attractive scenario of radiative electroweak symmetry breaking, the mass parameters μ_1^2 and μ_2^2 start out at the GUT scale with a universal positive value

$$\mu_1^2 = \mu_2^2 = m_0^2 + \mu^2,$$

where m_0 is the universal scalar mass. Thus the symmetry is not broken at this scale. However, in the RG evolution to the electroweak scale, the large Yukawa coupling of the top quark to H_2 , which gives the top its mass, also drives the mass-squared parameter μ_2^2 of H_2 negative (competing against the QCD coupling), while the absence of a large Yukawa in the down sector keeps the mass squared of H_1 positive. The above conditions are easily satisfied for a large range of initial conditions if $h \sim O(1)$, resulting in a very natural picture of electroweak symmetry breaking. This picture is essentially lost in the large $\tan \beta$ scenario, for the reasons discussed below.

Firstly, since all the Yukawa couplings are comparable, the two higgs doublets tend to run in the same way, so either both μ_1^2 and μ_2^2 stay positive at the electroweak scale and the symmetry does not break, or both become negative and the potential becomes

unbounded from below. The effects which differentiate between their running, such as differing hypercharge assignments, are small, and a poor replacement for the usual $h_t \gg h_b$ condition. Furthermore, an $O(1)$ splitting between h_t and $h_{b,\tau}$ here is still of little use since it is quickly reduced in importance by the proximity to the fixed point.

Secondly, even when the electroweak symmetry is broken, a large hierarchy of VEVs must be generated between the two similarly evolving Higgs fields. To see the implications of this, let us recall that the minimization of the tree level potential yields

$$\sin 2\beta = \frac{-2\mu_3^2}{\mu_1^2 + \mu_2^2}. \quad (2.1)$$

From here we see that large $\tan \beta$ will require μ_3^2 to be small in magnitude relative to $\mu_1^2 + \mu_2^2$. The second minimization condition is

$$\tan^2 \beta = \frac{\mu_1^2 + m_Z^2/2}{\mu_2^2 + m_Z^2/2}, \quad (2.2)$$

or, equivalently,

$$m_Z^2 = \frac{2(\mu_1^2 - \mu_2^2 \tan^2 \beta)}{\tan^2 \beta - 1} = \frac{2(m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta)}{\tan^2 \beta - 1} - 2\mu^2, \quad (2.3)$$

which in turn implies that for $\tan \beta \gg 1$ the solution we are looking for requires $\mu_2^2 \simeq -m_Z^2/2$, thus setting the scale. We, thus, see that a large hierarchy of VEVs requires the large hierarchy $\mu_3^2 \ll \mu_1^2 + \mu_2^2$. This, as is well known, implies a degree of fine tuning between some of the parameters in the Lagrangian. Knowledge of $m_{H_1}^2$ and $m_{H_2}^2$ at Q_0 determines $\mu^2(Q_0)$ from the approximate relation

$$\mu^2(Q_0) \simeq -m_{H_2}^2 - \frac{m_Z^2}{2} + \frac{m_{H_1}^2}{\tan^2 \beta} \quad (2.4)$$

and then $B = \frac{\mu_3^2}{\mu^2}$ is also determined. Note that $|\mu_3^2|$ has to be small at large values of $\tan \beta$, so that B must also be small. In numerical studies it has been found that electroweak symmetry works well for reasonably large h_t so long as h_b is neglected. Upon

its inclusion, the realization of electroweak symmetry breaking engenders fine tuning. The dependence of μ is somewhat complicated, although it may seem from eq.(2.1) that small μ gives large $\tan\beta$. On the other hand A (the trilinear soft SUSY breaking parameter) seems to be relatively less important in absolute terms (i.e., one does not obtain any significant restriction on its magnitude), although it does influence the running of the soft-parameters. In the NMSSM on the other hand, it plays an important role as we shall see in the next section. We note here that the MSSM spectrum is unchanged if $M_{1/2} \rightarrow -M_{1/2}$ (where $M_{1/2}$ is the universal gaugino mass) when $A = 0$.

The main features of the sparticle spectrum turn out to be governed by (a) strong $\mu - M_{1/2}$ correlation, (b) large values of $M_{1/2} \gtrsim 400$ GeV, and (c) $M_{1/2} \gtrsim m_0$ (universal soft SUSY breaking scalar mass). This implies small mixing in the chargino and neutralino sectors. The lightest supersymmetric particle is mainly a bino, with a mass $\simeq 0.4 M_{1/2}$. The second lightest neutralino and lightest chargino are winos and hence almost degenerate in mass. The heaviest neutralino and chargino are Dirac (pseudo-Dirac in the case of neutralino) particles, with masses approximately equal to the parameter $|\mu(Q_0)|$. An important tree level relation is obtained for large $\tan\beta > 10$, for which the tree-level value of the lightest CP-even higgs mass (m_h) is equal to m_Z , whenever the CP-odd boson mass (m_A) is larger than m_Z , while for $m_A \leq m_Z$, $m_h = m_A$. This tree level relation is approximately stable under radiative corrections, with the only difference that the range for which $m_h = m_A$ holds, extends to values of m_A somewhat larger than m_Z . Therefore, large values of $\tan\beta$ and values of CP-odd Higgs in the desired range imply $m_h < m_Z$. We note that large values of $\tan\beta$ and values CP-odd higgs mass $m_A < 70$ GeV are preferred to improve the agreement with the value of $R_b \equiv \frac{\Gamma(Z \rightarrow b\bar{b})}{\Gamma(Z \rightarrow \text{hadrons})}$ measured at LEP [16].

3 NMSSM at Large $\tan \beta$

The potential for the Higgs fields of the NMSSM can be obtained from eq.(1.2) and eq.(1.3) through a standard procedure [12]. The minimization conditions (evaluated at Q_0 after all the parameters are evolved via their one-loop RG equations down to this scale), that determine the soft SUSY breaking Higgs masses in terms of the other parameters, are

$$m_{H_1}^2 = -\lambda \frac{v_2}{v_1} s(A_\lambda + ks) - \lambda^2(v_2^2 + s^2) + \frac{1}{4}(g^2 + g'^2)(v_2^2 - v_1^2), \quad (3.1)$$

$$m_{H_2}^2 = -\lambda \frac{v_1}{v_2} s(A_\lambda + ks) - \lambda^2(v_1^2 + s^2) + \frac{1}{4}(g^2 + g'^2)(v_1^2 - v_2^2), \quad (3.2)$$

$$m_S^2 = -\lambda^2(v_1^2 + v_2^2) - 2k^2 s^2 - 2\lambda s v_1 v_2 - k A_k s - \frac{\lambda A_\lambda v_1 v_2}{s}. \quad (3.3)$$

The first two minimization conditions can be rewritten as:

$$\tan^2 \beta = \frac{m_Z^2/2 + m_{H_1}^2 + \lambda^2 s^2}{m_Z^2/2 + m_{H_2}^2 + \lambda^2 s^2}, \quad (3.4)$$

$$\sin 2\beta = \frac{(-2\lambda s)(A_\lambda + ks)}{m_{H_1}^2 + m_{H_2}^2 + \lambda^2(2s^2 + v^2)}. \quad (3.5)$$

[Our normalization is such that $v^2 \equiv v_1^2 + v_2^2 (= 174\text{GeV})^2$ and $m_Z^2 = \frac{1}{2}(g^2 + g'^2)v^2$, where g and g' are the gauge couplings of $SU(2)$ and $U(1)$, respectively.] These two equations give us some insight into the manner in which our solutions are likely to behave. Eq. (3.4) guarantees that $\tan \beta$ must lie between 1 and m_t/m_b . The proof, as in the case of the MSSM, relies once more on the RG equations that govern the behaviour of mass parameters and may be proved simply by reductio ad absurdum. For this purpose we need only consider the following equation expressing the momentum dependence of the difference of two supersymmetry breaking scalar mass parameters:

$$\frac{d}{dt}(m_{H_1}^2 - m_{H_2}^2) = \frac{1}{8\pi^2}(-3h_t^2 X_t + 3h_b^2 X_b + h_\tau^2 X_\tau) \quad (3.6)$$

where $t = \log(\mu)$, the logarithm of the momentum scale, and X_i , $i = t, b, \tau$, are combinations of scalar masses and trilinear couplings:

$$\begin{aligned} X_t &= m_{\tilde{Q}}^2 + m_{\tilde{t}}^2 + m_{H_2}^2 + A_t^2, \\ X_b &= m_{\tilde{Q}}^2 + m_{\tilde{b}}^2 + m_{H_1}^2 + A_b^2, \\ X_\tau &= m_{\tilde{L}}^2 + m_{\tilde{\tau}}^2 + m_{H_1}^2 + A_\tau^2. \end{aligned} \tag{3.7}$$

It must be noted that in order to prove that $\tan\beta > 1$ we neglect h_b and h_τ , and for proving $\tan\beta < m_t/m_b$ we retain them.

From eq.(3.4) it is clear that, as in MSSM, in order to have large $\tan\beta$ with $m_{H_1}^2$ and $m_{H_2}^2$ being essentially degenerate, because the top and bottom Yukawa couplings are comparable in the RG evolution, the denominator of the equation has to be small at the weak scale. This implies the fine tuning condition

$$m_{H_2}^2 + \lambda^2 s^2 \approx -m_Z^2/2. \tag{3.8}$$

From this condition it follows that the correspondence with the MSSM will occur in a certain well defined manner with the identification of λs with μ . Similarly, we must identify $A_\lambda + ks$ with B , the bilinear soft supersymmetry breaking parameter of the MSSM. We will show below that for large values of $\tan\beta$ this identification occurs in a novel manner, not generic to the model, say, for $\tan\beta \simeq 1$. For large values of $\tan\beta$, eq.(3.5) implies

$$A_\lambda \approx -ks \tag{3.9}$$

This is analogous to the condition $B \approx 0$ of the MSSM. The situation here is complicated because A_λ is not a parameter that is fixed at Q_0 but is present from the outset. This is the first of the fine tuning problems we encounter in the NMSSM.

A rearrangement of eq.(3.5) equation yields

$$\lambda s(A_\lambda + ks) = \tan\beta(-m_{H_2}^2 - \lambda^2 s^2) \frac{m_Z^2 (\tan\beta^2 - 1)}{2 (\tan\beta^2 + 1)} - \frac{\lambda^2 v^2 \sin 2\beta}{2}. \tag{3.10}$$

In this equation we can discard the last term when $\tan \beta$ is large. Then, with the identification of the appropriate parameters in terms of those of the MSSM as described earlier, we recover all the MSSM relations of the previous sections *without having to go through a limiting procedure* that is required in the general case. The third minimization condition, eq.(3.3), can be rewritten as:

$$m_S^2 = -\lambda^2 v^2 - 2A_\lambda^2 + \frac{\lambda v^2 \sin 2\beta A_\lambda}{k} + A_\lambda A_k + \frac{\lambda k v^2 \sin 2\beta}{2}. \quad (3.11)$$

In order to satisfy this condition, one must have large cancellations between the fourth and the first two terms, since the terms proportional to $\sin 2\beta$ are negligible. This requires that A_λ and A_k come out with the same sign and that their product be sufficiently large. It is this fine tuning condition that leads to problems with finding solutions with sufficiently small trilinear couplings in magnitude [13].

The starting point of our analysis of the NMSSM is the estimation of the scale M_X with the choice of the SUSY breaking scale $Q_0 \sim 1 \text{ TeV}$. For $\alpha_S(m_Z) = 0.12$, $Q_0 = 1 \text{ TeV}$ and $\alpha = 1/128$, we find upon integrating the one-loop beta functions, $M_X = 1.9 \times 10^{16} \text{ GeV}$, and the unified gauge coupling $\alpha_G(M_X) = 1/25.6$. We then choose a value for the unified Yukawa coupling h of $O(1)$. The free parameters of the model are the common gaugino mass ($M_{1/2}$), the common scalar mass (m_0), the common trilinear scalar coupling (A), and the two additional Yukawa couplings (λ , k), respectively. Note that our convention requires us to choose $\lambda > 0$ and $k < 0$ in order to conserve CP in the Yukawa sector of the model [13]. We also impose the constraint that $|A| < 3m_0$ [17] in order to guarantee the absence of electric-charge breaking vacua. In the case at hand this choice may have to be strengthened further due to the presence of large Yukawa couplings for the b-quark. The situation is considerably less restrictive when mild non-universality is allowed and, for instance, if strict Yukawa unification is relaxed. Given these uncertainties, we choose

to work with this constraint. We also compute the mass of the charged Higgs boson [12]

$$m_C^2 = m_W^2 - \lambda^2 v^2 - \lambda(A_\lambda + ks) \frac{2s}{\sin 2\beta}, \quad (3.12)$$

where m_W is the mass of the W-boson. We note that the radiative corrections to the charged Higgs mass are small for most of the parameter range, as in the case of MSSM [18]. The reason for this is that a global $SU(2) \times SU(2)$ symmetry [19] protects the charged Higgs mass from obtaining large radiative corrections. If this quantity were to come out to be negative, then the resulting vacuum would break electric-charge spontaneously and the corresponding point in the parameter space would be excluded.

Choosing a particular set of values for the input parameters $(M_{1/2}, m_0, A, \lambda, k)$, satisfying the above constraints, we calculate the crucial parameters $\tan \beta$, $r(\equiv s/v)$, A_λ and A_k and ks , that determine the physical spectrum, from their renormalization group evolution to Q_0 . We choose these input parameters in such a manner so as to ensure that we are in the neighborhood of a vacuum that breaks $SU(2) \times U(1)$ [13]. With the boundary condition (1.1) at the unification scale, we find $|M_{1/2}| \sim 500$ GeV for a wide range of values of $\lambda \sim 0.4 - 0.6$ with $k = -0.1$, leading to values of $\tan \beta \sim 60$. These magnitudes emerge when we choose λ in a manner that does not lead to very small values of k , and furthermore, we note that the solutions depend only on the ratio λ/k [13]. Once the sign of k is fixed [from requirements of CP invariance], a solution is found only when the sign of $M_{1/2}$ is negative. Phenomenological requirements enforce m_0 also to be rather large in order to guarantee that the lighter scalar tau be heavier than the lightest neutralino. The minimization condition (3.3) or its equivalent (3.11) enforces a large ratio $|A/m_0| \sim 3$. It then follows from eq.(3.9) that s must be rather large in comparison with m_Z , the only other scale in the problem. We note that here we are near the fixed point of λ , and as such we expect that this provides us with a lower bound on the singlet vacuum expectation value s . Essentially, this implies that in order

to have Yukawa unification (1.1) in the NMSSM, and the resulting large value of $\tan\beta$, the singlet vacuum expectation value must be large compared to the doublet vacuum expectation values. As we shall see in Sec. 4, these conclusions are borne out by the detailed numerical calculations.

We now analyze the implications of the large values of the singlet vacuum expectation value s , that arises in the NMSSM with eq. (1.1), on the mass spectra. In the basis $(\text{Im}H_1^0, \text{Im}H_2^0, \text{Im}S)$, with neutral Goldstone boson $G^0 = \cos\beta(\text{Im}H_1^0) - \sin\beta(\text{Im}H_2^0)$ and the orthogonal combination $A^0 = \cos\beta(\text{Im}H_1^0) + \sin\beta(\text{Im}H_2^0)$, the tree-level mass squared matrix for the two pseudoscalar neutral Higgs fields can be written as (in numerical calculations we take into account one-loop radiative corrections in the effective potential approximation)

$$M_P^2 = \begin{pmatrix} \frac{2\lambda s A_\Sigma}{\sin 2\beta} & \lambda v(A_\Sigma + 3ks) \\ \lambda v(A_\Sigma + 3ks) & -3ksA_k + \frac{\lambda v^2 \sin 2\beta}{2s}(A_\Sigma - 3ks) \end{pmatrix}, \quad (3.13)$$

where $A_\Sigma = -(A_\lambda + ks)$. The pseudoscalar masses are

$$m_{P_1, P_2}^2 = \frac{1}{2}[M_{22} + M_{11} \mp \sqrt{(M_{22} - M_{11})^2 + 4M_{12}^2}], \quad (3.14)$$

where

$$\begin{aligned} M_{22} \pm M_{11} &= \frac{\lambda v^2 \sin 2\beta}{2s}(A_\Sigma - 3ks) - 3ksA_k \pm \frac{2\lambda s A_\Sigma}{\sin 2\beta}, \\ M_{12} &= \lambda v(A_\Sigma + 3ks). \end{aligned} \quad (3.15)$$

The pseudoscalar eigenstates are described through the mixing angle γ given by:

$$\sin 2\gamma = \frac{-2M_{12}}{\sqrt{(M_{22} - M_{11})^2 + 4M_{12}^2}}. \quad (3.16)$$

For large values of s , the pseudoscalar Higgs masses are approximately given by

$$m_{P_1}^2 \simeq -3ksA_k, \quad m_{P_2}^2 \simeq \frac{2\lambda s A_\Sigma}{\sin 2\beta}. \quad (3.17)$$

This shows that the pseudoscalar P_1 is mainly $\text{Im}S$, whereas P_2 is a mixture of $\text{Im}H_1^0$ and $\text{Im}H_2^0$. Furthermore, for large values of $\tan\beta$, P_2 becomes very massive. Thus P_1 decouples from the spectrum because it is mainly a singlet and P_2 decouples from the spectrum because it is very massive simplifying that the pseudoscalar Higgs bosons would be impossible to produce in the limit of Yukawa unification. These qualitative features of the pseudoscalar spectrum survive the effects of radiative corrections. Here *we find a qualitative distinction with the corresponding pseudoscalar spectrum of the MSSM*. On the other hand the tree level the squared mass matrix, M_S^2 , for the neutral scalar Higgs bosons can be written as

$$\begin{pmatrix} m_Z^2 \cos^2 \beta + A_\Sigma s \tan \beta & -\lambda s A_\Sigma + \lambda^2 v^2 \sin 2\beta - \frac{m_Z^2 \sin 2\beta}{2} & \lambda v_2 (2\lambda s \cot \beta + ks - A_\Sigma) \\ -\lambda s A_\Sigma + \lambda^2 v^2 \sin 2\beta - \frac{m_Z^2 \sin 2\beta}{2} & m_Z^2 \sin^2 \beta + A_\Sigma s \cot \beta & \lambda v_1 (2\lambda s \tan \beta + ks - A_\Sigma) \\ \lambda v_2 (2\lambda s \cot \beta + ks - A_\Sigma) & \lambda v_1 (2\lambda s \tan \beta + ks - A_\Sigma) & (4k^2 s^2 + ksa_k) - \frac{\lambda A_\lambda v^2 \sin 2\beta}{2s} \end{pmatrix}. \quad (3.18)$$

We shall label the CP-even Higgs eigenstates of eq.(3.18) as S_1 , S_2 , and S_3 , respectively in order of increasing mass. It is not very illuminating to present complicated analytical expressions for them, and so we will defer the numerical results to Sec. 4. It turns out that in the limit of large $\tan\beta$, the lightest CP-even Higgs boson is almost a pure $\text{Re}(H_2^0)$, whereas the heaviest CP-even Higgs boson is predominantly the singlet, $\text{Re}S$. Furthermore, the lightest Higgs boson mass has an upper bound which is close to the upper bound in the MSSM in the same limit. This is a consequence of the fact that the upper bound on the lightest Higgs mass in the two models, including radiative corrections, differs by a term that is proportional to $\sin 2\beta$, and is, therefore, small [20].

The neutralino mass matrix (M_{χ^0}) in the Non Minimal Supersymmetric Standard model can be written as [21] :

$$\begin{pmatrix} M_1 & 0 & -m_Z \cos \beta \sin \theta_w & m_Z \sin \beta \sin \theta_w & 0 \\ 0 & M_2 & m_Z \cos \beta \cos \theta_w & -m_Z \sin \beta \cos \theta_w & 0 \\ -m_Z \cos \beta \sin \theta_w & m_Z \cos \beta \cos \theta_w & 0 & \lambda s & \lambda v \sin \beta \\ m_Z \sin \beta \sin \theta_w & -m_Z \sin \beta \cos \theta_w & \lambda s & 0 & \lambda v \cos \beta \\ 0 & 0 & \lambda v \sin \beta & \lambda v \cos \beta & 2ks \end{pmatrix}, \quad (3.19)$$

where M_1 and M_2 are the masses of the $U(1)_Y$ and $SU(2)_L$ gauginos ($\lambda'; \lambda^a, a = 1, 2, 3$), respectively, and where we have chosen the basis $(-i\lambda', -i\lambda^3, \Psi_{H_1^1}, \Psi_{H_2^2}, \Psi_S)$. The grand unification condition leads to the mass relation $M_1 = (5/3)M_2 \tan^2 \theta_W$. The gluino mass is M_3 ; the gluino does not mix with the rest of the neutralinos. Because neutralinos are Majorana particles, the mass matrix (3.19) is in general complex symmetric, and hence can be diagonalized by only one unitary matrix N , i.e., $N^* M_{\chi^0} N^{-1} = M_{\chi^0}^D$. We shall label the neutralino masses in the ascending order (of magnitude) as $M_{\chi_i^0}$, $i = 1, \dots, 5$.

In the limit of large s , we have the approximate expressions for the (magnitude of) neutralino masses in increasing order:

$$M_1, M_2, \lambda s, \lambda s, 2ks, \quad (3.20)$$

with the lightest neutralino being a predominantly a bino, which satisfies the simple mass relation

$$M_1 \simeq \frac{\alpha_1(Q_0)}{\alpha_G} M_{1/2} \simeq 0.45 M_{1/2}. \quad (3.21)$$

All the neutralinos with the exception of the heaviest one have negligible singlet component; the singlet decouples from the remainder of the spectrum. We note that two of the neutralinos are nearly degenerate, thus leading to a pseudo-Dirac neutralino, as in MSSM, in the limit of large $\tan \beta$.

The chargino mass matrix M_{χ^\pm} is given by:

$$\begin{pmatrix} M_2 & \sqrt{2}m_W \sin \beta \\ \sqrt{2}m_W \cos \beta & -\lambda s \end{pmatrix} \quad (3.22)$$

and is same as in MSSM with the role of μ played by λs . The masses and the composition of the charginos are obtained by diagonalizing the matrix (3.22) via the biunitary transformation $U M_{\chi^\pm} V^{-1} = M_{\chi^\pm}^D$, where U and V are 2×2 matrices which diagonalize the hermetian matrices $M_{\chi^\pm} M_{\chi^\pm}^\dagger$ and $M_{\chi^\pm}^\dagger M_{\chi^\pm}$, respectively. Similarly, the masses of other sparticles are same as in MSSM with μ replaced by λs .

4 Numerical Results and Discussion

Having described the NMSSM with large $\tan\beta$ in detail in the previous section, we now turn to obtaining the particle spectrum of the model numerically.

Writing down the coupled set of the RG equations for the 24 parameters of the model [22, 23], and including the contributions of h_b and h_τ [13], the parameter space of the model is scanned by taking values of the input parameters $(M_{1/2}, m_0, A, h, \lambda, k)$ at M_X which are evolved down to low energies Q_0 to obtain the values of the parameters $\tan\beta$, $r \equiv s/v$, A_λ , A_k , ks . We also evolve the values of the soft masses appearing on the left hand sides of eqs. (3.1) – (3.3), and compare their values to the combinations of the parameters appearing on the right hand sides of these equations as obtained from the RG evolution. The input parameters are chosen so as to satisfy the constraints described in the previous section, namely the absence of electric charge breaking vacua, the charged Higgs mass squared remaining positive, and the $SU(2) \times U(1)$ breaking minimum being energetically favorable. It turns out that the first of the minimization conditions, eq.(3.1), is the one which is most sensitive to the choice of initial conditions reflecting the fine tuning discussed in Sec. 3. The parameters are chosen so as to study r_1 , r_2 and r_3 , which are defined as the difference between the left and right hand sides of the three minimization eqs. (3.1) – (3.2) divided by the right hand side of each of these equations, and study the change in sign that these suffer as the parameters are varied. There are enormous difficulties in trying to achieve a simultaneous solution to $r_i = 0$, $i = 1, 2, 3$. In particular $r_3 = 0$ requires the presence of values for $|A|/m_0$ of almost 3 or more. This requirement has a profound impact on the particle spectrum of the model. In particular, r , the ratio of the singlet to the doublet vacuum expectation value persistently remains large for the choice of parameters considered with large $\tan\beta$, corresponding to the VEV of s being of the order of 10 TeV. This is substantially different from the situation when

$\tan\beta \simeq 1$ [12]. We note that the singlet vacuum expectation value is not constrained by the experimental data.

In Fig. 1, we show a typical evolution of the three soft SUSY breaking mass parameters $m_{H_1}^2$, $m_{H_2}^2$ and m_S^2 from M_X down to the low scale Q_0 with a choice of parameters such that all constraints are satisfied, and we are in the neighbourhood of an $SU(2) \times U(1)$ breaking vacuum. We note that because of the possibility of large value of $m_t(m_t) = 181$ GeV, we have a large value for h so that the Yukawa couplings dominate over the gauge couplings in the evolution of these parameters. This in turn forces the mass parameters to remain large at large momentum scales compared to their values at smaller momentum scales.

In supersymmetric theories with R-parity conservation, the lightest supersymmetric particle generally turns out to be the lightest neutralino. In the NMSSM the neutralino mass matrix is given by eq.(3.19) and its general properties are discussed in [21]. The parameters that determine the mass matrix are $\lambda, k, s, \tan\beta, M_1$ and M_2 . Choosing the input parameters at M_X so that all the constraints of the Sec. 3 are satisfied, we obtain values for these parameters at Q_0 and then the neutralino mass matrix may be evaluated numerically. The chargino mass matrix eq.(3.22) may also be evaluated in a similar manner.

One result of this procedure is shown in Fig. 2, where we plot the lightest neutralino and chargino masses for a specific choice of input parameters of Table 1, as obtained from RG evolution, as a function of the top quark mass. We have found from our scan of the parameter space, that the lightest neutralino is almost a pure bino in the limit of large $\tan\beta$. Furthermore, all other neutralinos except the heaviest one have a negligible singlet component, indicating that the singlet completely decouples from the lighter neutralino spectrum. These properties of the neutralino spectrum are shown in the second and third

columns of Table 2. The mass of the lightest neutralino in this case is determined by the simple mass relation for the bino $M_1 \simeq \left(\frac{\alpha_1(M_G)}{\alpha_G}\right) M_{1/2} \simeq 0.45 M_{1/2}$. The masses of the heavier neutralinos lie in the range of $0.5 - 1$ TeV. Furthermore, the lightest chargino mass bears a relation to $M_{1/2}$ similar to the neutralino relation, with α_1 in eq. (3.21) replaced by α_2 , reflecting that it is primarily a charged wino. This is seen from the fourth, fifth and sixth columns of Table 2. The heavier chargino mass is found to be $\simeq 1$ TeV. We further note that two of the neutralinos are nearly degenerate, and lead to a pseudo-Dirac neutralino. Also the second lightest neutralino is primarily a wino and degenerate in mass with the lightest chargino. These characteristics are similar to those found in MSSM. The gluino mass is found to be 1.6 TeV for the choice of parameters of Fig. 2 and follows from a relation similar to eq. (12) with α_1 replaced by α_S .

We now come to the spectrum of CP-even Higgs bosons of the model. In order to understand the quantitative features of the results we have obtained for the lightest CP-even Higgs boson, we need to go into some detail regarding the actual choices of parameters entering the computation and the correlations between the various elements of the spectrum. We have already reviewed the corresponding situation in the case of the MSSM in Sec. 2, and here we will discuss the comparison of the NMSSM with that of the MSSM. In Fig. 3, we plot for typical and reasonable values of the input parameters, in the region where the vacuum is expected to lie, the mass of the lightest CP-even Higgs boson as a function of the top-quark mass $m_t(m_t)$ in the range that is most favoured under these boundary conditions [3, 6, 7, 8, 24]. The choice of parameters here is closely related to the family of solutions studied extensively in Ref. [13], and would serve as a typical example of the numbers we have explored. In the MSSM when the mass of its unique CP-odd Higgs boson $m_A \gg m_Z$, the substantive part of the radiative correction is picked up by the lighter of the CP-even bosons, h^0 . As m_A approaches m_Z , the radiative corrections

are shifted to the heavier of the CP-even Higgs bosons, H^0 . Such a feature is observed here as well: for those choices of parameters in Table 1 that yield a somewhat smaller m_{P_1} (the mass of lightest CP-odd Higgs boson in NMSSM), we find that the radiative corrections to the lightest CP-even Higgs, S_1 , are smaller. We note, however, that the CP-odd Higgs bosons here are always massive in the limit of large $\tan\beta$ in contrast to the situation that prevails in the MSSM, where m_A even in the vicinity of m_Z is plausible [6]. Due to the complexity of the system under investigation and the difficulty in controlling the numerical choice of the parameter λ for a given h , with all other parameters held fixed, we do not know how precisely close the choice of parameters of Table 1 are to a genuine ground state. Furthermore, we note that the clarity with which the correlations have been observed between m_A and m_{h^0} in MSSM do not have a simple parallel here due to the presence of a larger number of physical states. A more precise, albeit prohibitively time consuming, determination could then ensure that the spurious wobble seen in Fig. 3 is eliminated, and would establish a more reliable correlation between increasing h and the rise of the mass of S_1 and the correlations with m_{P_1} . Furthermore, a refinement of the choice of parameters, based on the minimization of the one-loop effective potential, could stabilize the figures presented here.

We note that the lightest Higgs bosons mass ~ 130 GeV for a wide range of parameters which nearly saturates the upper bound of 140 GeV [13], and lies in the same range as in MSSM with large $\tan\beta$. This is a consequence of the largeness of $\tan\beta$: the contribution to the tree level mass which depends on the trilinear couplings λ is small, being proportional to $\sin^2 2\beta$, so that the upper bound on the lightest Higgs mass reduces to the corresponding upper bound in MSSM when the appropriate identification of the parameters is performed [20]. We also note that the upper bound on the lightest Higgs mass depends only logarithmically on r , and hence on the singlet vacuum expectation value s ,

in the limit of large r , which, therefore decouples from the bound [25]. Furthermore, the lightest Higgs boson is almost a pure doublet Higgs (Re H_2^0), with the singlet component being less than 1% in the entire range of parameters considered. It is only the second heavier CP even Higgs boson S_2 that is predominantly a singlet. Its mass ranges between 740 GeV and 2.3 TeV. The heaviest CP even Higgs boson S_3 is again predominantly a doublet Higgs (Re H_1^0) with its mass varying between 4 – 6 TeV. This implies that all the CP-even Higgs bosons, except the lightest one, decouple from the spectrum. These features of the spectrum of the CP-even Higgs bosons of NMSSM for large values of $\tan \beta$ are clearly seen from Table 3, where we show the mass and composition of S_i for a wide range of input parameters. The results presented above, that the lightest Higgs boson is almost purely a doublet Higgs at large $\tan \beta$, are in contrast to the situation with low values of $\tan \beta$, where the lightest CP-even Higgs boson contains a large admixture of the gauge singlet field S [12, 26, 27].

In Fig. 4 we plot for the typical values of the input parameters of Table 1 the mass of the lightest CP-odd Higgs boson as a function of $m_t(m_t)$. We note from Table 3 that both CP-odd Higgs bosons P_1 and P_2 are heavy, their masses being in the range of 2 TeV and 6 TeV, respectively. Also, the lightest CP-odd state is predominately a Higgs singlet, thereby effectively decoupling from the rest of the spectrum. The charged Higgs boson mass m_C lies, for most of the cases that we have studied, in the range 1 – 2 TeV.

In order to discuss the features of the the sfermion spectrum, we first recall some of the features of the spectrum of the MSSM. In the MSSM it has been observed [6] that the presence of large Yukawa couplings for the b-quark as well as the τ lepton, as well as the presence of large trilinear couplings, could lead to the lighter of the scalar τ 's tending to become lighter than the lightest neutralino, which is the most favoured candidate in such models for the lightest supersymmetric particle. In particular, in order to overcome

cosmological constraints for given values of $M_{1/2}$, lower bounds on m_0 were found to emerge. In turn, increasing m_0 implies ever decreasing m_A (clear correlations have been described for the case of $A = 0$ in Ref. [6]) thus leading to further constraints on the parameter space of the MSSM. For large values of $\tan\beta$ the competing tendencies between the lighter scalar tau mass and m_A play an important role in MSSM in establishing a lower bound ~ 450 GeV on $M_{1/2}$. Given the complexity of the system of equations, it has not been possible to extract similar lower bounds on $M_{1/2}$ in NMSSM. Nevertheless, in Ref. [13] the intimate link between the ground states of the two models has been established and a much more sophisticated and time consuming analysis of the present model is also likely to yield a lower bound that is unlikely to be very different from the one obtained in MSSM. As a result, in confining ourselves to numbers of this magnitude and higher, we find a heavy spectrum. More recently [9] further experimental constraints on MSSM have been taken into account resulting in an extension of the minimal assumptions at M_X by including non-universality for scalar masses. Indeed, in the present analysis similar problems have been encountered with some of the choice of parameters studied in Ref. [13], with $m_{\tilde{\tau}_1}$ tending to lie below the mass of the lightest neutralino due to the persistent presence of large Yukawa couplings and more so due to the large trilinear couplings dictated by eq.(3.3). Nevertheless, given the fact that the present work minimizes the tree-level potential and that the violations of cosmological constraints are not serious, in that minor adjustments of $|A|/m_0$ solve this problem efficiently, we consider the regions of the parameter space we have explored to be reasonable. Furthermore, it could be that the extension of minimal boundary conditions along the lines of Ref. [9] could provide alternative and elegant solutions to this problem, while preserving the existence of relatively light scalar τ 's as a prediction of the unification of Yukawa couplings in the NMSSM as well as in the MSSM.

The heaviest sfermions in the spectrum of NMSSM, as in MSSM, are the scalar quarks, which tend to be much heavier, in the TeV range. The $SO(10)$ property that the scalar b-quarks are as massive as the scalar top-quarks is preserved in the NMSSM.

To summarize, through a detailed analysis of NMSSM presented here, we have shown that all the particles, except the lightest CP - even Higgs boson, implied by supersymmetry are heavy for large values of $\tan\beta$. The gauge singlet field S decouples, both from the lightest Higgs boson as well as the neutralinos. The LSP of the model continues to be, as in MSSM, the lightest neutralino that is primarily a bino in composition, with the lighter scalar τ having a mass in the neighbourhood of the LSP mass. The remainder of the spectrum tends to be heavy, from 1 to a few TeV. We note that the NMSSM in the large $\tan\beta$ regime rests on a delicately hinged system of equations and constraints. Although it provides a good testing ground for the stability of the predictions of the MSSM, in practice it deserves great care in its treatment.

Acknowledgements: The research of BA is supported by the Swiss National Science Foundation. PNP thanks the Alexander von Humboldt-Stiftung and Universität Kaiserslautern, especially Prof. H. J. W. Müller-Kirsten, for support while this work was completed. The work of PNP is supported by the Department of Science and Technology, India under Grant No. SP/S2/K-17/94.

References

- [1] J. Ellis, S. Kelly and D. V. Nanopoulos. Phys. Lett. B260 (1991) 131; U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B260 (1991) 447; P. Langacker and M. X. Luo, Phys. Rev. D44 (1991)817
- [2] For reviews, see H. P. Nilles, Phys. Rep. 110 (1984) 1; R. Arnowitt and P. Nath, Lectures at the Swieca School, Campos do Jordao, Brazil, 1993, CTP-TAMU-52/93 and NUB-TH-3073-93
- [3] B. Ananthanarayan, G. Lazarides and Q. Shafi, Phys. Rev. D44 (1991) 1613
- [4] J. Gasser and H. Leutwyler, Phys. Rep. 87 (1982) 77
- [5] F. Abe et al., Phys. Rev. D52 (1995) 2605; S. Abachi et al., Phys. Rev. Lett. 74 (1995) 2632
- [6] B. Ananthanarayan, G. Lazarides and Q. Shafi, Phys. Lett. B300 (1993) 245; B. Ananthanarayan, Q. Shafi and X-M. Wang, Phys. Rev. D50 (1994) 5980
- [7] M. Olechowski and S. Pokorski, Phys. Lett. B214 (1988) 393; M. Olechowski and S. Pokorski, Nucl. Phys. B404 (1994) 590; E. G. Floratos, G. K. Leontaris and S. Lola, Nucl. Phys. B452 (1995) 471
- [8] G. Anderson et al., Phys. Rev. D49 (1994) 3660; L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D50 (1994) 7048; M. Carena et al. Nucl. Phys. B426 (1994) 269; T. Blazek, S. Raby and S. Pokorski, OHSTPY-HEP-T-95-007, hep-ph/9504364
- [9] D. Matalliotakis and H. P. Nilles, Nucl. Phys. B435 (1995) 115; M. Olechowski and S. Pokorski, Phys. Lett. B344 (1995) 201; H. Murayama, M. Olechowski and S. Pokorski, hep-ph/9510327

- [10] H. Arason et al., Phys. Rev. Lett. 67 (1991) 2933; M. Carena et al., Nucl. Phys. B369 (1992) 33; V. Barger, M. S. Berger and P. Ohmann, Phys. Rev. D 47 (1993) 1093; B. Ananthanarayan, K. S. Babu and Q. Shafi, Nucl. Phys. B428 (1994) 19
- [11] P. Fayet, Nucl. Phys. B90 (1975) 104; R. K. Kaul and P. Majumdar, Nucl. Phys. B199 (1982) 36; R. Barbieri, S. Ferrara and C. A. Savoy, Phys. Lett. B119 (1982) 343; H-P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B120 (1983) 346; J. M. Frere, D. R. T. Jones and S. Raby, Nucl. Phys. B222 (1983) 11; J-P. Derendinger and C. A. Savoy, Nucl. Phys. B237 (1984) 307
- [12] J. Ellis et al., Phys. Rev. D 39 (1989) 844
- [13] B. Ananthanarayan and P. N. Pandita, Phys. Lett. B353 (1995) 70; B. Ananthanarayan and P. N. Pandita, BUTP-95/38, KL-TH-95/25 (to appear in Phys. Lett. B)
- [14] See, e.g., Ref.[2]; H.E. Haber and G.L. Kane, Phys. Rep. 117 (1985) 75
- [15] L. Ibañez and G. G. Ross, Phys. Lett. 110B (1982) 215; H. P. Nilles, Phys. Lett. B115 (1982) 193
- [16] P. Antilogus et al., [LEP Electroweak Working Group], LEPEWWG/95-02 (1995); For a review in the context of supersymmetry see, e.g., C. E. M. Wagner, Talk presented at SUSY '95, Palasiau, France (May 1995), CERN-Th/95-261
- [17] G. Gamberini, R. Ridolfi and F. Zwirner, Nucl. Phys. B331 (1990) 331
- [18] A Brignole, Phys. Lett. B277 (1992) 313; M. A. Diaz and H. E. Haber, Phys. Rev. D45 (1992) 4246
- [19] H. E. Haber and A. Pomarol, Phys. Lett. B 302 (1993) 435

- [20] P. N. Pandita, Phys. Lett. B318 (1993) 338; Z. Phys. C59 (1993); U. Ellwanger, Phys. Lett. B303 (1993) 271; T. Elliot, S. F. King and P. L. White, Phys. Lett. B305 (1993) 71; Phys. Rev. D49 (1994) 2435; S. F. King and P. L. White, Phys. Rev. D52 (1995) 4183
- [21] P. N. Pandita, Phys. Rev. D50 (1994) 571; Z. Phys. C 63 (1994) 659
- [22] See, e.g., J-P. Derendinger and C. A. Savoy in Ref. [11]
- [23] N. K. Falck, Z. Phys. C30 (1986) 247
- [24] B. C. Allanach and S. F. King, Phys. Lett. B328 (1994) 360
- [25] P. N. Pandita in Ref. [20]; D. Comelli and C. Verzegnassi, Phys. Rev. D47 (1993) R764
- [26] U. Ellwanger et al., Phys. Lett. B315 (1993) 331
- [27] T. Elliot et al., Phys. Lett. B305 (1993) 71

Table Captions

Table 1 Sample of values of input parameters ($M_{1/2}$, m_0 , A , h , k , λ) and the computed values of different parameters ($m_t(m_t)$, $\tan\beta$, r , A_λ , A_k and ks). All mass parameters are in units of GeV.

Table 2 Masses and compositions of neutralino χ_i^0 and chargino χ_i^\pm states for the values of input parameters of Table 1. All masses are in GeV.

Table 3 Masses and compositions of the CP-even (S_i) and CP-odd (P_i) Higgs bosons. The composition of CP-even Higgs bosons is in terms of the basis (ReH_1^0 , ReH_2^0 , ReS), whereas the basis for CP-odd Higgs bosons is (A^0 , ImS). All masses are in GeV.

Figure Captions

Fig. 1 The evolution of soft supersymmetry breaking mass parameters from the grand unified scale M_X to Q_0 defined in the text. The input parameters are $M_{1/2} = -700$, m_0 and $A = 1600$ (all in GeV). The other parameters are $h = 1.5$, $\lambda = 0.40$ and $k = -0.10$. The associated value of the top quark mass is 181 GeV.

Fig. 2 The lightest neutralino and chargino masses as a function of $m_t(m_t)$. The input parameters are $M_{1/2} = -700$, $m_0 = 800$, $A = 2200$ (all in GeV), with the remaining parameters varied to guarantee a solution.

Fig. 3 The lightest CP-even Higgs boson mass as a function of m_t . The range of parameters is as in Fig. 2.

Fig. 4 The lightest CP-odd Higgs boson mass as a function of m_t . The range of parameters is as in Fig. 2.

#	$M_{1/2}$	m_0	A	h	λ	k	$m_t(m_t)$	$\tan \beta$	r	A_λ	A_k	ks
1	-700	800	2200	0.75	0.1	-0.1	170.5	54	85	908	2131	-1453
2	-700	800	2200	1.00	0.2	-0.1	176.5	58	58	807	2100	-975
3	-700	800	2200	1.25	0.3	-0.1	179.6	60	49	745	2070	-807
4	-700	800	2200	1.50	0.4	-0.1	181.4	62	44	703	2045	-717
5	-700	800	2200	1.75	0.4	-0.1	182.5	63	51	698	2067	-835
6	-700	800	2200	2.00	0.5	-0.1	183.3	64	47	675	2048	-755
7	-700	800	2200	2.25	0.6	-0.1	183.8	64	44	657	2032	-700

Table 1

#	$M_{\chi_i^0}$	N_{ij}	$M_{\chi_i^\pm}$	V_{ij}	U_{ij}
1	−309	0.99, −0.01, −0.06, 0.02, 0.00	579	0.99, 0.15	0.97, 0.22
	−579	0.02, 0.98, 0.16, −0.11, 0.00			
	877	0.03, −0.04, 0.70, 0.70, 0.00			
	−887	0.05, −0.20, 0.70, −0.70, 0.00			
	−2906	0.00, 0.00, 0.00, 0.00, 0.99	886	−0.15, 0.99	−0.22, 0.97
2	−309	0.99, −0.01, −0.05, 0.02, 0.00	581	0.99, 0.12	0.97, 0.22
	−580	0.01, 0.99, 0.14, −0.09, 0.00			
	941	0.02, −0.04, 0.70, 0.70, 0.00			
	−949	0.05, −0.16, 0.70, −0.70, 0.01			
	−1950	0.00, 0.00, −0.01, 0.00, 0.99	949	−0.12, 0.99	−0.22, 0.97
3	−309	0.99, −0.06, −0.05, 0.02, 0.00	581	0.99, 0.10	0.98, 0.19
	−581	0.01, 0.97, 0.14, −0.09, 0.00			
	936	0.02, −0.04, 0.70, 0.70, 0.00			
	−944	0.05, −0.16, 0.69, −0.70, 0.00			
	−1613	0.00, 0.00, −0.02, 0.01, 0.99	945	−0.10, 0.99	−0.19, 0.98
4	−309	0.99, 0.00, −0.05, 0.02, 0.00	582	0.99, 0.12	0.99, 0.07
	−582	0.01, 0.99, 0.12, −0.07, 0.00			
	1001	0.02, −0.03, 0.70, 0.70, 0.00			
	−1007	0.04, −0.14, 0.70, −0.7, 0.04			
	−1436	0.00, 0.00, −0.03, 0.02, 0.99	1008	−0.12, 0.99	−0.07, 0.99
5	−309	0.99, 0.00, −0.05, 0.02, 0.00	582	0.99, 0.10	0.99, 0.17
	−582	0.01, 0.99, 0.13, −0.08, 0.00			
	979	0.02, −0.03, 0.70, 0.70, 0.00			
	−986	0.05, −0.14, 0.70, −0.7, 0.00			
	−1669	−0.33, 0.00, −0.02, 0.00, 0.99	1018	−0.10, 0.99	−0.17, 0.98
6	−309	0.99, 0.00, −0.05, 0.02, 0.00	581	0.99, 0.10	0.94, 0.35
	−582	0.01, 0.99, 0.13, −0.08, 0.00			
	978	0.02, −0.04, 0.70, 0.70, 0.00			
	−984	0.05, −0.14, 0.70, −0.7, 0.00			
	−1509	0.00, 0.00, −0.02, 0.01, 0.99	985	−0.10, 0.99	−0.35, 0.94
7	−309	0.99, 0.00, −0.05, 0.01, 0.00	582	0.99, 0.10	0.98, 0.17
	−582	0.01, 0.99, 0.12, −0.07, 0.00			
	998	0.02, −0.03, 0.70, 0.70, 0.00			
	−1004	0.05, −0.14, 0.70, −0.7, 0.04			
	−1402	0.00, 0.00, −0.03, 0.03, 0.99	1004	−0.10, 0.99	−0.17, 0.98

Table 2

#	m_{S_i}	Composition	m_{P_i}	Composition
1	131	$-0.02, 0.99, 0.00$	3048	$0.00, 1.00$
	2314	$0.00, 0.00, 0.99$		
	6039	$0.99, -0.02, 0.00$		
2	132	$0.02, 0.99, -0.02$	2479	$0.00, 1.00$
	1326	$0.00, 0.02, 0.99$		
	4822	$0.99, -0.02, 0.00$		
3	130	$0.02, 0.99, -0.05$	2238	$0.00, 1.00$
	967	$0.00, 0.05, 0.99$		
	4342	$0.99, -0.17, 0.00$		
4	119	$0.02, 0.99, -0.09$	2099	$0.00, 1.00$
	774	$0.00, -0.9, 0.99$		
	4297	$0.99, -0.02, 0.00$		
5	133	$0.02, 0.99, -0.04$	2275	$0.00, 1.00$
	1031	$0.00, -0.04, 0.99$		
	5116	$0.99, -0.02, 0.00$		
6	128	$0.02, 0.99, -0.07$	2153	$0.00, 1.00$
	857	$0.00, 0.07, 0.99$		
	4823	$0.99, -0.02, 0.00$		
7	119	$0.02, 0.99, -0.01$	2067	$0.00, 1.00$
	739	$0.00, 0.10, 0.99$		
	4644	$0.99, -0.02, 0.00$		

Table 3

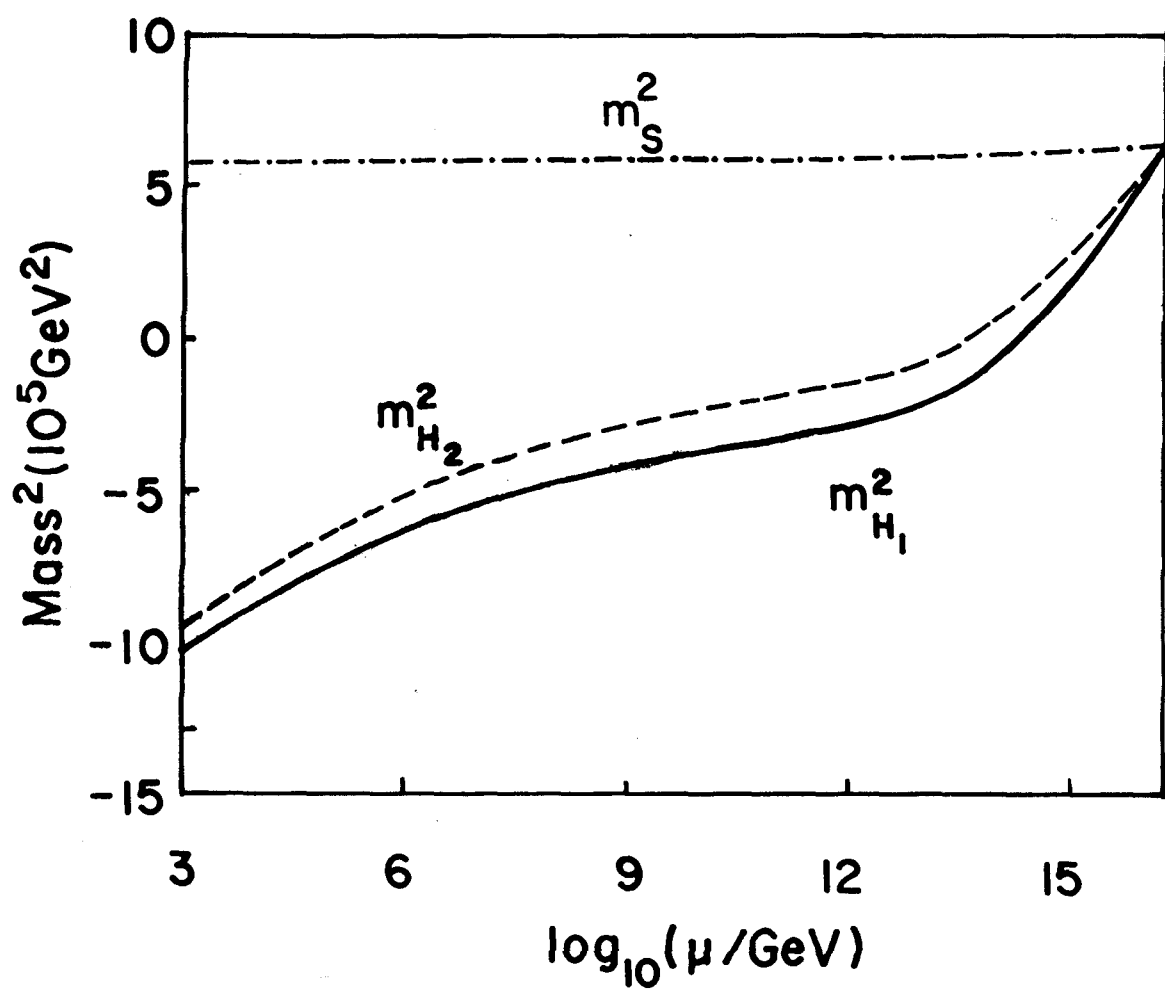


Fig.1

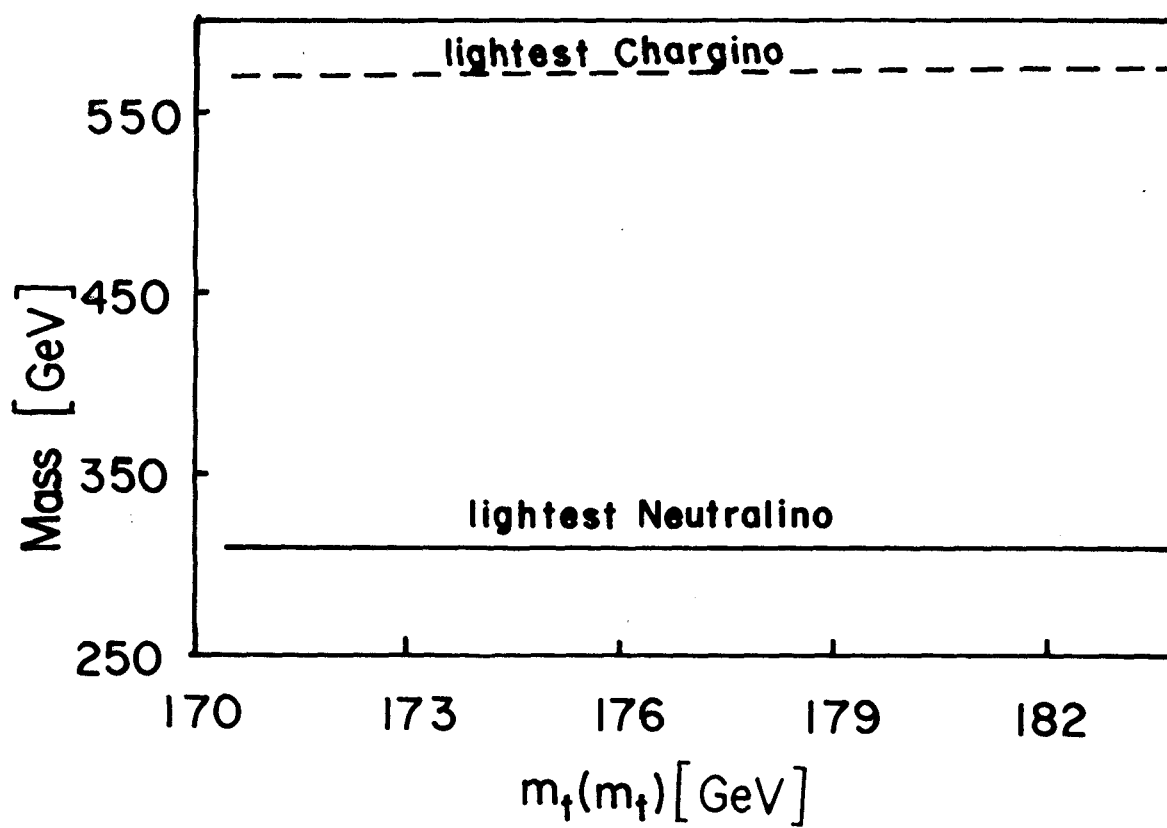


Fig.2

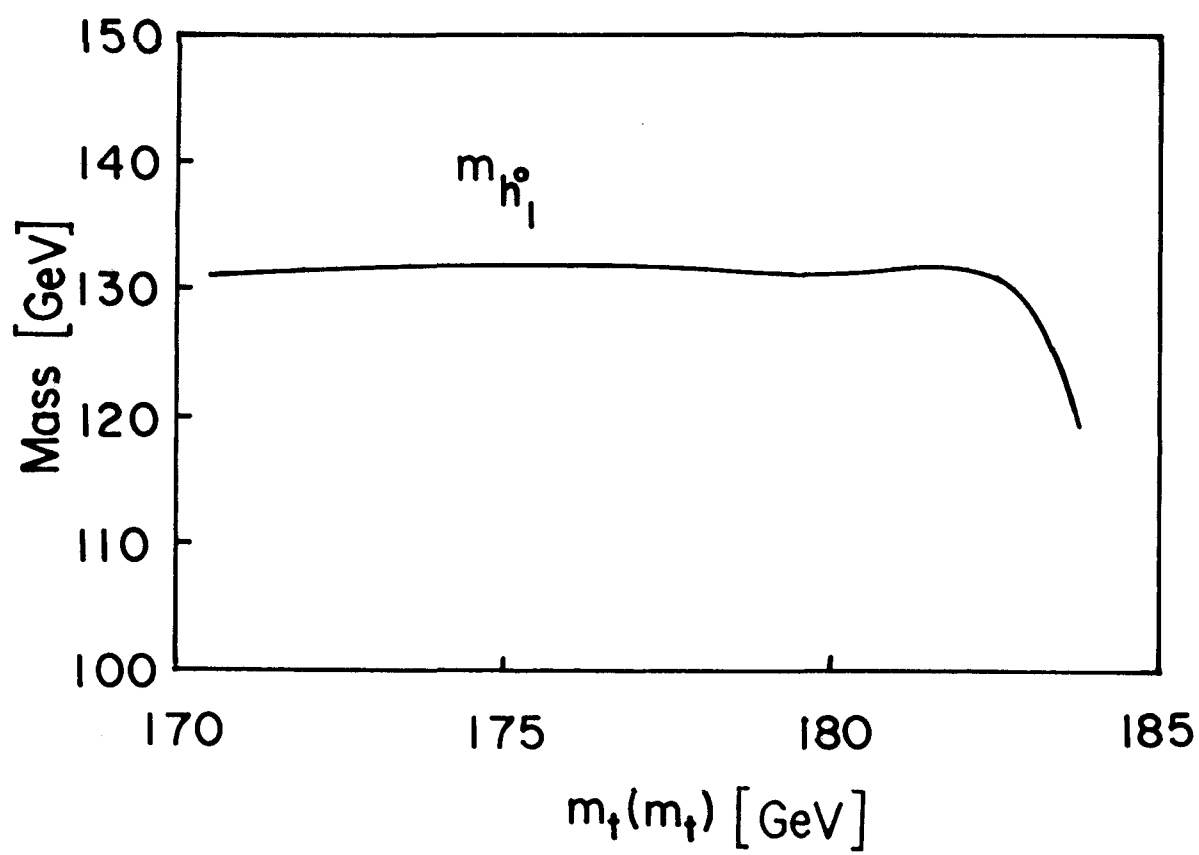


Fig.3

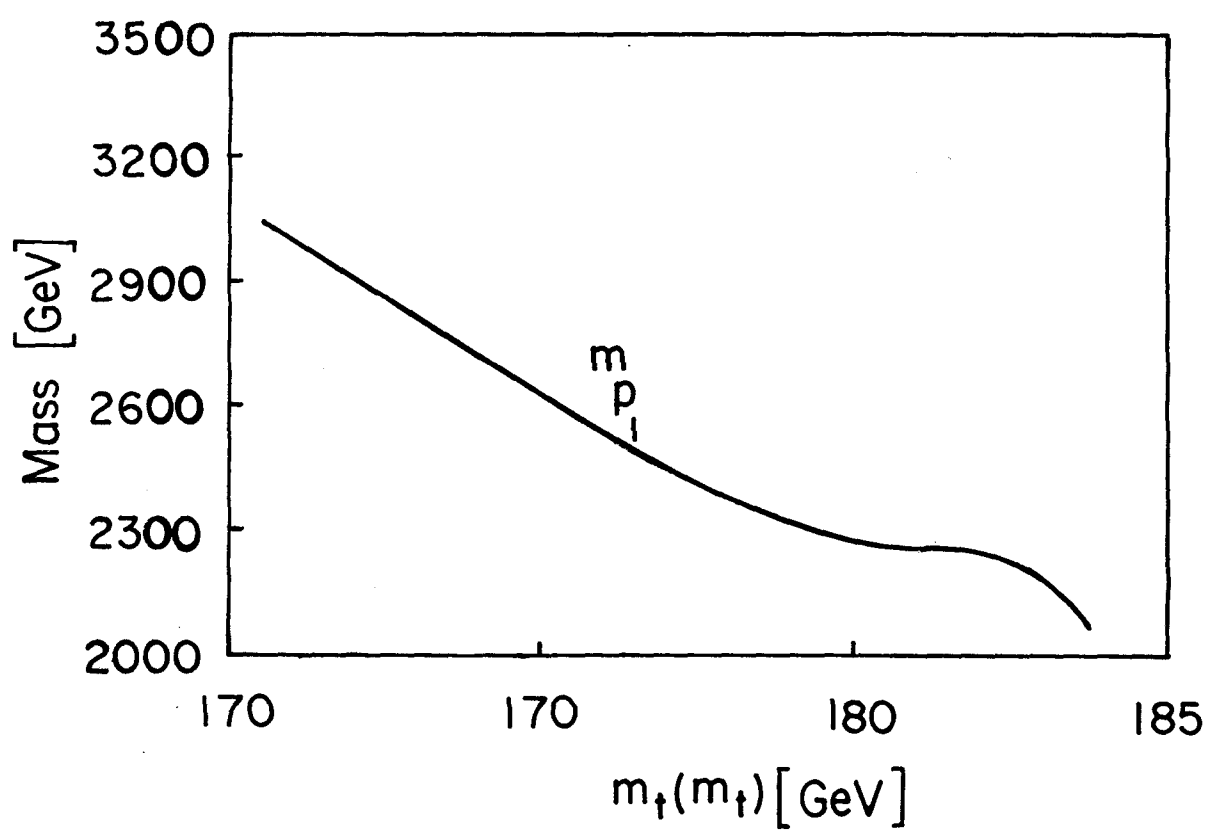


Fig.4